

Effect of Dynamical Cosmological Constant in presence of Modified Chaplygin Gas for Accelerating Universe

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(Dated: February 1, 2008)

In this paper we have considered the Universe to be filled with Modified Gas and the Cosmological Constant Λ to be time-dependent with or without the Gravitational Constant G to be time-dependent. We have considered various phenomenological models for Λ , viz., $\Lambda \propto \rho$, $\Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \frac{\ddot{a}}{a}$. Using these models it is possible to show the accelerated expansion of the Universe at the present epoch. Also we have shown the natures of G and Λ over the total age of the Universe. Using the statefinder parameters we have shown the diagrammatical representation of the evolution of the Universe starting from radiation era to Λ CDM model.

PACS numbers:

I. INTRODUCTION

There are two parameters, the cosmological constant Λ and the gravitational constant G , present in Einstein's field equations. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein's field equations. In an evolving Universe, it appears natural to look at this "constant" as a function of time. Numerous suggestions based on different arguments have been proposed in the past few decades in which G varies with time [1]. Dirac [2] proposed a theory with variable G motivated by the occurrence of large numbers discovered by Weyl, Eddington and Dirac himself.

It is widely believed that the value of Λ was large during the early stages of evolution and strongly influenced its expansion, whereas its present value is incredibly small [3]. Several authors [4] have advocated a variable Λ in the framework of Einstein's theory to account for this fact. Λ as a function of time has also been considered in various variable G theories in different contexts [5]. For these variations, the energy-momentum tensor of matter leaves the form of the Einstein's field equations unchanged.

In attempt to modify the General Theory of Relativity, Al-Rawaf and Taha [6] related the cosmological constant to the Ricci Scalar \mathcal{R} . This is written as a built-in-cosmological constant, i.e., $\Lambda \propto \mathcal{R}$. Since the Ricci Scalar contains a term of the form $\frac{\ddot{a}}{a}$, one adopts this variation for Λ . We parameterized this as $\Lambda \propto \frac{\ddot{a}}{a}$. Similarly, we have chosen another two forms for Λ : $\Lambda \propto \rho$ and $\Lambda \propto \frac{\dot{a}^2}{a^2}$; where ρ is the energy density.

Recent observations of the luminosity of type Ia Supernovae indicate [7, 8] an accelerated expansion of the Universe and lead to the search for a new type of matter which violates the strong energy condition i.e., $\rho + 3p < 0$. The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as a *dark energy*. There are different candidates to play the role of the dark energy. The type of dark energy represented by a scalar field is often called Quintessence. The simplest candidate for dark energy is Cosmological Constant Λ . In particular one can try another type of dark energy, the so-called Chaplygin gas which obeys an equation of state like [9] $p = -B/\rho$, ($B > 0$), where p and ρ are respectively the pressure and energy density. Subsequently the above equation was modified to the form [10] $p = -B/\rho^\alpha$, $0 \leq \alpha \leq 1$. There are some works on Modified Chaplygin Gas obeying equation of state [11, 12]

$$p = A\rho - B/\rho^\alpha, (A > 0) \quad (1)$$

In this work we have considered a cosmological model for the cosmological constant of the forms: $\Lambda \propto \frac{\ddot{a}}{a}$, $\Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \rho$ in presence of Modified Chaplygin Gas, with or without the variation of Gravitational

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Constant G .

In 2003, V. Sahni et al [13] introduced a pair of parameters $\{r, s\}$, called *statefinder* parameters. The trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. The above statefinder diagnostic pair has the following form:

$$r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \quad (2)$$

where $H (= \frac{\dot{a}}{a})$ and $q (= -\frac{a\ddot{a}}{\dot{a}^2})$ are the Hubble parameter and the deceleration parameter respectively. The new feature of the statefinder is that it involves the third derivative of the cosmological radius. These parameters are dimensionless and allow us to characterize the properties of dark energy. Trajectories in the $\{r, s\}$ plane corresponding to different cosmological models, for example Λ CDM model diagrams correspond to the fixed point $s = 0, r = 1$.

The paper is organized as follows: Section II deals with Einstein field equations in presence of dynamic cosmological constant Λ . In sections III and IV we have considered different Λ -dependent models without and with variable gravitational constant G respectively and shown various stages of the evolution of the Universe for these models using statefinder parameters. We have taken some particular values of the proportionality constants for the graphical representations. We have discussed this particularization of the constants and their physical consequences in section V.

II. EINSTEIN FIELD EQUATIONS AND DYNAMIC COSMOLOGICAL CONSTANT

We consider the spherically symmetric FRW metric,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (3)$$

where $a(t)$ is the scale factor and k is the curvature scalar with values 0, 1 and -1 for respectively flat, closed and open models of the Universe. The Einstein field equations for a spatially flat Universe (i.e., taking $k = 0$) with a time-dependent cosmological constant $\Lambda(t)$ are given by (choosing $c = 1$),

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\rho + \Lambda(t) \quad (4)$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi Gp + \Lambda(t) \quad (5)$$

where ρ and p are the energy density and isotropic pressure respectively.

Let us choose modified chaplygin gas with equation of state given by equation (1).

Here, we consider the phenomenological models for $\Lambda(t)$ of the forms $\Lambda \propto \rho$, $\Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \frac{\ddot{a}}{a}$. First we will consider G to be constant and try to find out the solutions for density ρ and the scale factor $a(t)$ and hence study the cosmological models in terms of the statefinder parameters r, s . Secondly we will consider G to be variable as well and study the various phases of the Universe represented by the models.

III. MODELS KEEPING G CONSTANT AND Λ VARIABLE

Taking G to be constant and Λ to be time dependent, the energy conservation equation is,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\frac{\dot{\Lambda}}{8\pi G} \quad (6)$$

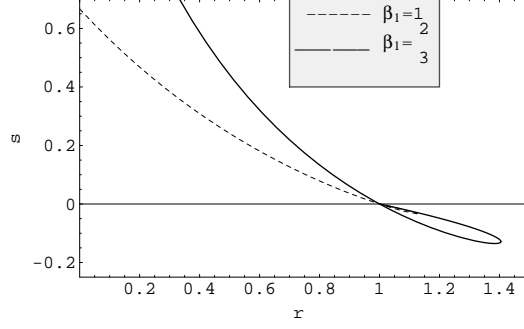


Fig.1

Fig. 1 shows the variation of s against r for different values of $\beta_1 = 1, \frac{2}{3}$ respectively and for $\alpha = 1, A = 1/3$ and $8\pi G = 1$.

A. Model with $\Lambda \propto \rho$

Here we consider

$$\Lambda = \beta_1 \rho \quad (7)$$

where β_1 is a constant.

Equation (7) together with equations (1) and (6) yield the solution for ρ to be,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G + \beta_1}}} \right)^{\frac{1}{1+\alpha}} \quad (8)$$

where C is an arbitrary constant.

Substituting equation (7) and (8) in equation (4), we get the solution for the scale factor $a(t)$ as,

$$a^{f_1 f_2} \sqrt{8\pi G + \beta_1} {}_2F_1[f_2, f_2, 1 + f_2, -\frac{a^{f_1 B}}{C(1+A)}] = 4\sqrt{3}(1+A)G\pi C^{f_2} t \quad (9)$$

where $f_1 = \frac{24(1+A)(1+\alpha)\pi G}{8\pi G + \beta_1}$ and $f_2 = \frac{1}{2(1+\alpha)}$. Hence, for small values of $a(t)$, we have, $\rho \simeq \left(\frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G + \beta_1}}} \right)^{\frac{1}{1+\alpha}}$ which is very large and the equation of state (1) reduces to $p \simeq A\rho$. Again for large values of $a(t)$, we get $\rho \simeq \left(\frac{B}{1+A} \right)^{\frac{1}{1+\alpha}}$ and $p \simeq -\left(\frac{B}{1+A} \right)^{\frac{1}{1+\alpha}}$, i.e., $p \simeq -\rho$ which coincides with the result obtained for MCG with $\beta_1 = 0$ [12].

Using equations (2), (4) and (6) we get,

$$r = 1 + \frac{36\pi G(1+y)[8\pi G\{A(1+\alpha) - y\alpha\} - \beta_1]}{(8\pi G + \beta_1)^2} \quad \text{and} \quad s = \frac{8\pi G(1+y)[8\pi G\{A(1+\alpha) - y\alpha\} - \beta_1]}{(8\pi G + \beta_1)(8\pi Gy - \beta_1)} \quad (10)$$

where $y = \frac{p}{\rho}$ which can be further reduced to a single relation between r and s . Now $q = -\frac{\ddot{a}}{aH^2} = \frac{8\pi G(1+3y)-2\beta_1}{2(8\pi G + \beta_1)}$. Therefore for acceleration $q < 0 \Rightarrow y < \frac{\beta_1}{12\pi G} - \frac{1}{3}$. Also for the present epoch $q = -\frac{1}{2} \Rightarrow y = \frac{1}{3}(\frac{3\beta_1}{8\pi G} - 2)$.

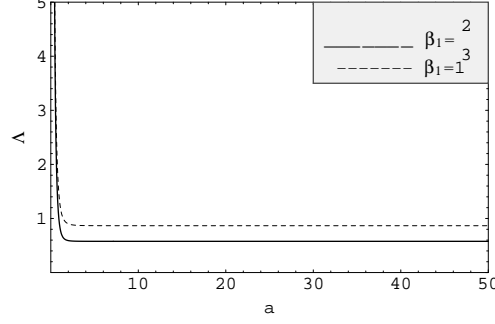


Fig.2

Fig. 2 shows the variation of Λ against $a(t)$ for different values of $\beta_1 = 1, \frac{2}{3}$ respectively and for $\alpha = 1, A = 1/3, 8\pi G = 1, B = 1, C = 1$.

If we assume that the present Universe is dust filled, we have $y = 0$, i.e., $\beta_1 = \frac{16\pi G}{3}$. Taking $8\pi G = 1$ we get the best fit value to be $\beta_1 = \frac{2}{3}$, which gives $r = 1$ (choosing $A = \frac{1}{3}, \alpha = 1$) for the present Universe. That means the dark energy responsible for the the present acceleration is nothing but Λ . Also $A = 1, \alpha = 1$ and $\beta_1 = \frac{2}{3}$ give $r = 2.16$ for the present time. For this case $\beta_1 > 3$ gives non-feasible solutions in the sense that the present values of y , i.e., $\frac{2}{\rho}$ becomes too large. For $\beta_1 = 1$, we get the present value of y to be $\frac{1}{3}$, but again $r = 1$. In either of the above cases we get accelerating expansion of the Universe. These can be represented diagrammatically in the r, s plane. This is shown in figure 1 (taking $A = \frac{1}{3}, \alpha = 1, \beta_1 = 1, \frac{2}{3}, 8\pi G = 1$ and $A = 1, \alpha = 1, \beta_1 = 1, \frac{2}{3}, 8\pi G = 1$). Figure 1 represents the evolution of the Universe starting from radiation era to Λ CDM model. Here we get a discontinuity at $\beta_1 = -8\pi G$.

Again for this model

$$\Lambda = \beta_1 \left(\frac{B}{1+A} + \frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G + \beta_1}}} \right)^{\frac{1}{1+\alpha}} \quad (11)$$

Variation of $\Lambda(t)$ against $a(t)$ is shown in figure 2 for different choices of β_1 , which represents that regardless the values of $\beta_2, \Lambda(t)$, i.e., the effect of the cosmological constant decreases with time.

B. Model with $\Lambda \propto H^2$

Choosing

$$\Lambda(t) = \beta_2 H^2 \quad (12)$$

where β_2 is a constant and proceeding as above, we obtain the solutions for $\rho, a(t), \Lambda$ as,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{(3-\beta_2)(1+A)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad (13)$$

$$a^{f_1 f_2} {}_2F_1[f_2, f_2, 1 + f_2, -\frac{a^{f_1 B}}{C(1+A)}] = \sqrt{2\pi G} \sqrt{3 - \beta_2} (1+A) C^{f_2} t \quad (14)$$

where $f_1 = (3 - \beta_2)(1 + A)(1 + \alpha)$ and $f_2 = \frac{1}{2(1+\alpha)}$

$$\Lambda = \frac{8\pi G \beta_2}{3 - \beta_2} \left(\frac{B}{1+A} + \frac{C}{a^{(3-\beta_2)(1+A)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad (15)$$

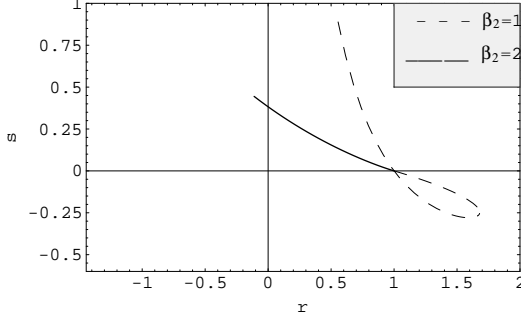


Fig.3

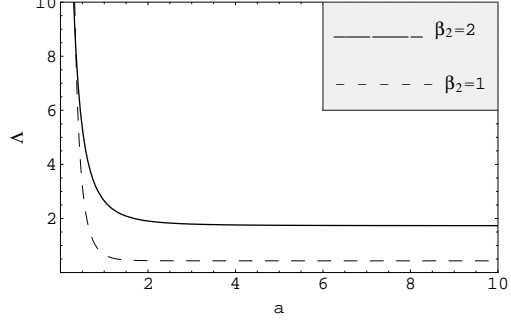


Fig.4

Fig. 3 shows the variation of s against r for different values of $\beta_2 = 1, 2$ and for $\alpha = 1, A = 1/3, 8\pi G = 1$. Fig. 4 shows the variation of Λ against $a(t)$ for different values of $\beta_2 = 1, 2$ and for $\alpha = 1, A = 1/3, 8\pi G = 1$.

Here for $\beta_2 < 3$ we can check the consistency of the result by showing $p \simeq A\rho$ at small values of $a(t)$ and $p = -\rho$ for large values of $a(t)$. But if we take $\beta_2 > 3$ we get opposite results which contradict our previous notions of the nature of the equation of state (1). Again for $\beta_2 = 3$, we get only Λ CDM point, i.e., we get a discontinuity. Therefore, we restrict our choice for β_2 in this case to be $\beta_2 < 3$.

Computing the state-finder parameters given by equation (2), we get the equations for r and s to be,

$$r = 1 + \frac{(3 - \beta_2)(1 + y)[\{A(1 + \alpha) - y\alpha\}(3 - \beta_2) - \beta_2]}{2} \quad \text{and} \quad s = \frac{(3 - \beta_2)(1 + y)[\{A(1 + \alpha) - y\alpha\}(3 - \beta_2) - \beta_2]}{3\{(3 - \beta_2)y - \beta_2\}} \quad (16)$$

(where $y = \frac{p}{\rho}$), which can still be resolved into a single relation and can be plotted in the r, s plane. Here $q = \frac{1}{2}[(3 - \beta_2)y - (\beta_2 - 1)]$. Hence the Universe will accelerate if $q < 0 \Rightarrow y < \frac{\beta_2 - 1}{3 - \beta_2}$. Again for the present Universe $q = -\frac{1}{2} \Rightarrow y = \frac{\beta_2 - 2}{3 - \beta_2}$. Assuming the present Universe to be dust dominated, i.e., $y = 0$ we get the best fit value for β_2 to be 2. Taking $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1, \beta_2 = 2$ and $y = 0$ (i.e. dust dominated present Universe) we get the present value to be $r = 1/3$, also the same values with $\beta_2 = 1$ gives the present values to be $y = -\frac{1}{2}, r = \frac{5}{3}$. This is shown in figure 3 ($A = \frac{1}{3}, \alpha = 1, \beta_2 = 1$ and $2, 8\pi G = 1$), which explains the evolution of the Universe from radiation era to Λ CDM model. Again variation of Λ against time is shown in figure 4, where we can see that Λ decreases with time for whatever the value of β_2 be.

C. Model with $\Lambda \propto \frac{\ddot{a}}{a}$

Taking

$$\Lambda = \beta_3 \frac{\ddot{a}}{a} \quad (17)$$

(where β_3 is a constant), and proceeding as above we get a relation for ρ as,

$$\rho^{(\frac{2}{1+A} - \beta_3)} \left(1 + A - \frac{B}{\rho^{\alpha+1}}\right)^{(\frac{2}{(1+A)(1+\alpha)} - \beta_3)} = \frac{C}{a^{2(3-\beta_3)}} \quad (18)$$

Unlike the previous two cases here we get a far more restricted solution. Here the only choice of β_3 for which we get the feasible solution satisfying $p \simeq A\rho$ for small values of $a(t)$ and $p \simeq -\rho$ for large values of $a(t)$ is

$$\beta_3 < \frac{2}{(1+A)(1+\alpha)} \quad \text{or} \quad \beta_3 > 3 \quad (19)$$

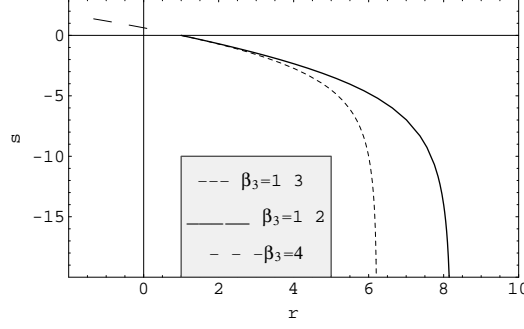


Fig.5

Fig. 5 shows the variation of s against r for different values of $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1, \beta_3 = \frac{1}{2}, 4$ and $\frac{1}{3}$

Again since $q = -\frac{\ddot{a}}{aH^2} = -\frac{\Lambda}{\beta_3 H^2} = \frac{4\pi G(\rho+3p)}{(3-\beta_3)H^2} = \frac{4\pi G(\rho+3p)}{(3-\beta_3)H^2}$, $\beta_3 > 3$ implies $q < 0$ without even violating the energy-condition $\rho + 3p \geq 0$. Although $\beta_3 < \frac{2}{(1+A)(1+\alpha)}$ causes the acceleration of the Universe violating the energy-condition. Taking $q = -\frac{1}{2}$ for the present epoch, we obtain $y = \frac{(\beta_3-4)}{(6-\beta_3)}$. Hence the present epoch is dust filled if $\beta_3 = 4$ and thus giving the present value of r to be $-\frac{1}{7}$ for $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1$. On using relation (19), ρ and therefore a, Λ cannot be expressed in an open form. We can rather derive a solution for Λ in terms of p, ρ as,

$$\Lambda = \frac{4\pi G\beta_3}{\beta_3 - 3}(\rho + 3p) \quad (20)$$

Using equations (2) we get the statefinder parameters as,

$$r = 1 - \frac{(1+y)(\beta_3-3)[\beta_3 + (\beta_3+6)x]}{(\beta_3 + \beta_3 x - 2)(\beta_3 + \beta_3 y - 2)} \quad \text{and} \quad s = \frac{2(1+y)(\beta_3-3)[\beta_3 + (\beta_3+6)x]}{[\beta_3 + (\beta_3+6)y][\beta_3 + \beta_3 x - 2]} \quad (21)$$

where $y = \frac{p}{\rho}$ and $x = \frac{\partial p}{\partial \rho}$, i.e., $x = A(1+\alpha) - y\alpha$ [from equation (1)].

Eliminating y between the equations (21), we get a single relation of r and s , which can be represented diagrammatically in the r, s plane (figure 5). Here we have taken $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1, \beta_3 = \frac{1}{2}, 4$ and $\frac{1}{3}$, combining two cases. Taking $\beta_3 = \frac{1}{2}, \frac{1}{3}$ we can explain the evolution of the Universe starting from $\frac{p}{\rho} = -\frac{1}{3}$ to Λ CDM model and $\beta_3 = 4$ explains the evolution of the Universe starting from radiation era to $y = -\frac{1}{3}$, as seen from the expression for q . Considering the present epoch to be dust-dominated, the present value of r is given for $\beta_3 = 4$ to be $-\frac{1}{7}$. As follows, the former two cases cannot give the present value of r , as $y = 0 > -\frac{1}{3}$ for the present epoch. Here we have an infinite discontinuity at $\frac{p}{\rho} = -\frac{1}{3}$, i.e., when $\rho + 3p = 0$. Also since we do not get a closed form of ρ here, it is difficult to plot Λ against the scale factor $a(t)$.

IV. MODELS WITH Λ AND G BOTH VARIABLE

Now we consider G as well as Λ to be variable. With this the equations (3), (4), (5) yield the conservation laws as,

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (22)$$

and

$$\dot{\Lambda} + 8\pi\dot{G}\rho = 0 \quad (23)$$

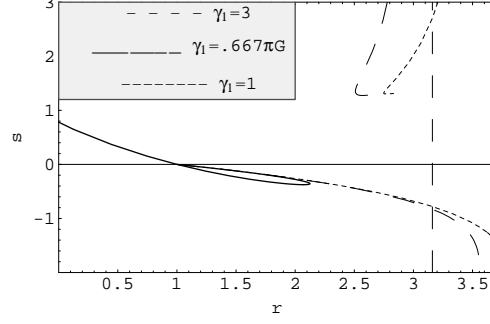


Fig.6

Fig. 6 shows the variation of s against r for different values of $\gamma_1 = 1, 3$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_1 = 1$.

Now we study the various phases of the Universe represented by these models.

Equation (22) together with equation (1) yield the solution for ρ as,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad (24)$$

where C is an arbitrary constant. This result is consistent with the results already obtained [12].

A. Model with $\Lambda \propto \rho$

Here we consider

$$\Lambda = \gamma_1 \rho \quad (25)$$

where γ_1 is a constant.

Equation (22), (23) and (25) give,

$$G = C_1 - \frac{\gamma_1}{8\pi} \log \rho \quad (26)$$

where C_1 is a constant and ρ is given by equation (24).

Using equations (2), (4), (23) and (26), we get,

$$\begin{aligned} G &= C_1 + \frac{\gamma_1(1+\alpha)}{8\pi} \log\left(\frac{B}{A-y}\right) \\ r &= 1 + \frac{9(1+y)[8\pi G\{A(1+\alpha)-y\alpha\}-\gamma_1(1+y)]}{2(8\pi G+\gamma_1)} \\ s &= \frac{(1+y)[8\pi G\{A(1+\alpha)-y\alpha\}-\gamma_1(1+y)]}{(8\pi G y - \gamma_1)} \end{aligned} \quad (27)$$

where $y = \frac{p}{\rho}$.

Equation (27) cannot be resolved to get a single relation between r and s , rather we obtain a parametric relation between the same with $y = \frac{p}{\rho}$ as the parameter. This can be represented diagrammatically in the r, s plane, which is shown in figure 6 taking $\gamma_1 = 1, 3$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_1 = 1$. Now

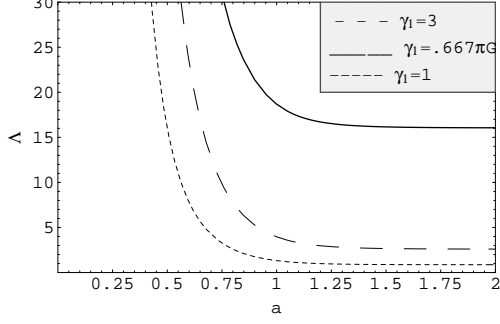


Fig.7

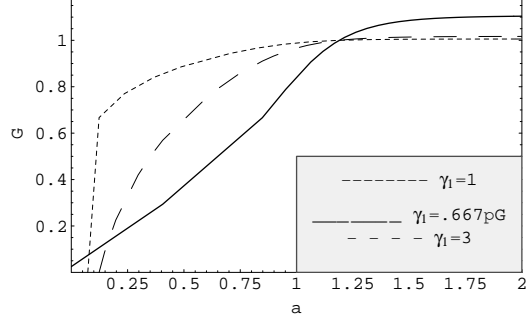


Fig.8

Fig. 7 shows the variation of Λ against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1, A = 1/3, C_1 = 1$. Fig. 8 shows the variation of G against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1, A = 1/3, C_1 = 1, B = 1, C = 1$.

$q = \frac{4\pi G(1+3y)-\gamma_1}{8\pi G+\gamma_1}$. Taking into account that $q = -\frac{1}{2}$ for the present epoch, we get $y = \frac{\gamma_1}{8\pi G} - \frac{2}{3}$. Therefore, for the present dust-dominated era $y = 0$ and $\gamma_1 = \frac{16\pi G}{3}$. Hence for the This models represents the Universe starting from the radiation era to Λ CDM model. Again figure 7 represents the variation of Λ against the scale factor $a(t)$ with $\gamma_1 = 1, 3, 3.5$ and figure 8 represents the variation of G against the scale factor $a(t)$. These figures show that for this particular phenomenological model of Λ, G starting from very low initial value increases largely and becomes constant after a certain period of time, whereas Λ starting from a very large decreases largely to reach a very low value and becomes constant.

B. Model with $\Lambda \propto H^2$

We consider

$$\Lambda = \gamma_2 H^2 \quad (28)$$

Proceeding as above we get,

$$\Lambda = 8\pi G \frac{\gamma_2}{3 - \gamma_2} \rho \quad (29)$$

where γ_2 is a constant.

Solving equation (22), (23) and (29) we get,

$$G = \frac{C_2}{\rho^{\frac{\gamma_2}{3}}} \quad (30)$$

where C_2 is a constant.

Using equations (2), (4) and (23), we find the state-finder parameters as,

$$r = 1 + \frac{(1+y)(3-\gamma_2)[3\{A(1+\alpha) - y\alpha\} - (1+y)\gamma_2]}{2} \quad (31)$$

$$s = \frac{(1+y)(3-\gamma_2)[3\{A(1+\alpha) - y\alpha\} - (1+y)\gamma_2]}{(3-\gamma_2)y - \gamma_2} \quad (32)$$

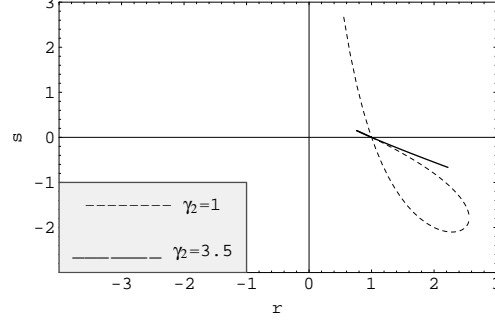


Fig.9

Fig. 9 shows the variation of s against r for different values of $\gamma_2 = 1$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_2 = 1, C = 1$.

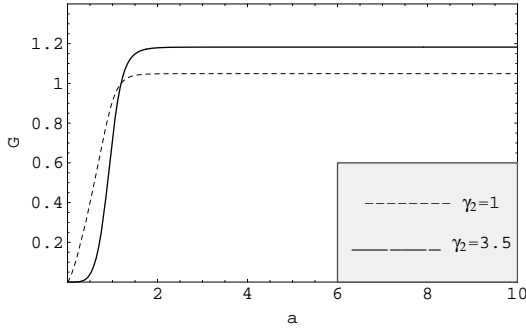


Fig.10

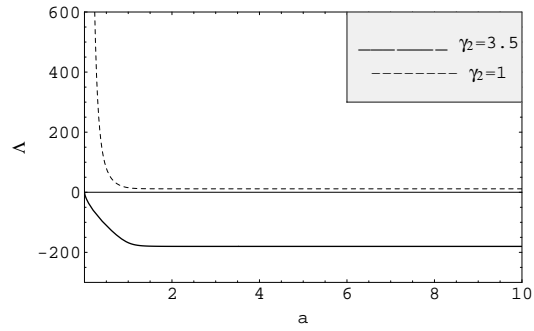


Fig.11

Fig. 10 shows the variation of G against $a(t)$ for different values of $\gamma_1 = 1, 3.5$ and for $\alpha = 1, A = 1/3, C_1 = 1, C = 1, B = 1$. Fig. 8 shows the variation of Λ against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1, A = 1/3, C_2 = 1, B = 1, C = 1$.

where $y = \frac{p}{\rho}$.

Now $q = \frac{1}{2}[(3 - \gamma_2)y - (\gamma_2 - 1)]$. These equations can further be resolved into a single relation of r and s , which can be plotted diagrammatically in the r, s plane. Here we get a discontinuity at $\gamma_2 = 3$. We have plotted these values in the r, s plane taking $\gamma_2 = 1$ and 3.5 in figure 9 ($A = \frac{1}{3}, \alpha = 1$). This case explains the present acceleration of the Universe, starting from radiation era to Λ CDM model.

Also figures 10 and 11 show respectively the variation of G and Λ against the scale factor for the same values of the constants. Here also like the previous case G starting from a very low initial value increases largely and then continues to be constant near unity. On the other hand Λ starting from a large value decreases largely and continues to be constant after a certain period of time.

C. Model with $\Lambda \propto \frac{\ddot{a}}{a}$

Here we consider

$$\Lambda = \gamma_3 \frac{\ddot{a}}{a} \quad (33)$$

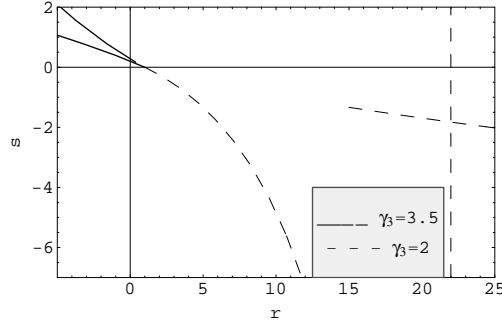


Fig.12

Fig. 12 shows the variation of s against r for different values of $\gamma_2 = 2$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_3 = 1, C = 1$.

where γ_3 is a constant.

Using equation (33) in equations (4) and (5), we get,

$$\Lambda = -\frac{4\pi G\gamma_3}{3-\gamma_3}(\rho + 3p) \quad (34)$$

Also, G can be solved to be,

$$G = C_3 [\rho^{\frac{1+3A}{2-\gamma_3(1+A)}} \{2 - \gamma_3(1+A) + \frac{B\gamma_3}{\rho^{\alpha+1}}\} \{-\frac{3\alpha}{\gamma_3(1+\alpha)} + \frac{1+3A}{(1+\alpha)(2-\gamma_3(1+A))}\}]^{\frac{\gamma_3}{3}} \quad (35)$$

Using equations (2), (4), (22), (23), we find,

$$r = 1 + \frac{(1+y)(3-\gamma_3)[6\{A(1+\alpha) - y\alpha\} + \gamma_3(1+y)]}{[2-\gamma_3(1+y)]^2} \quad (36)$$

$$s = \frac{2(1+y)(3-\gamma_3)[6\{A(1+\alpha) - y\alpha\} + \gamma_3(1+y)]}{3[2-\gamma_3(1+y)][\gamma_3 + (\gamma_3+6)y]} \quad (37)$$

where $y = \frac{p}{\rho}$ and C_3 is a constant. Equations (36) and (37) can further be resolved to get one single relation between r and s and plotted diagrammatically taking $\gamma_3 = 2$ and 3.5 (figure 12). Since deceleration parameter $q = -\frac{\ddot{a}}{aH^2} = -\frac{\lambda}{\gamma_3 H^2} = \frac{4\pi G}{(3-\gamma_3)H^2}$, is negative in the present epoch, we get $3 - \gamma_3 < 0$, i.e., $\gamma_3 > 3$. Also for $\gamma_3 = 3$ we get discontinuity. Both the models represent the phases of the Universe starting from radiation era to Λ CDM model. Again G and Λ can be plotted against a (figures 13 and 14 respectively). Unlike the previous cases this model an opposite nature of G and Λ , as G decreases with time and Λ increases with time.

V. DISCUSSION

Here we have considered three phenomenological models of Λ , with or without keeping G to be constant. Keeping G constant we always get accelerated expansion of the Universe. For the first case, i.e., $\Lambda \propto \rho$ or more precisely, $\Lambda = \beta_1 \rho$, for particular choices of the constants we get that the dark energy responsible for the present acceleration is nothing but Λ . Also the density parameter of the Universe for this case is given by, $\Omega_m^{\beta_1} = \frac{8\pi G\rho}{3H^2} = \frac{8\pi G}{8\pi G + \beta_1}$ and the vacuum density parameter is $\Omega_\Lambda^{\beta_1} = \frac{\Lambda}{3H^2} = \frac{\beta_1}{8\pi G + \beta_1}$, so that $\Omega_{total} = \Omega_m + \Omega_\Lambda = \Omega_m^{\beta_1} + \Omega_\Lambda^{\beta_1} = 1$. Also for $\Lambda \propto H^2$, i.e., $\Lambda = \beta_2 H^2$, the density parameter and vacuum density parameter are given by, $\Omega_m^{\beta_2} = \frac{3-\beta_2}{3}$ and $\Omega_\Lambda^{\beta_2} = \frac{\beta_2}{3}$ respectively, so that $\Omega_{total} = \Omega_m^{\beta_2} + \Omega_\Lambda^{\beta_2} = 1$. Again

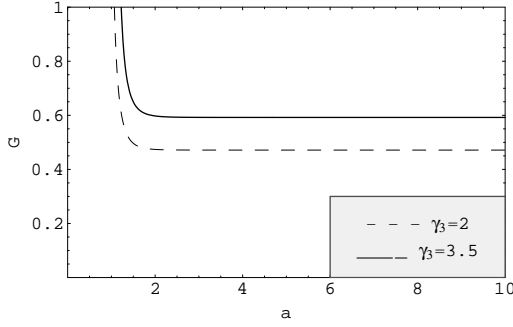


Fig.13

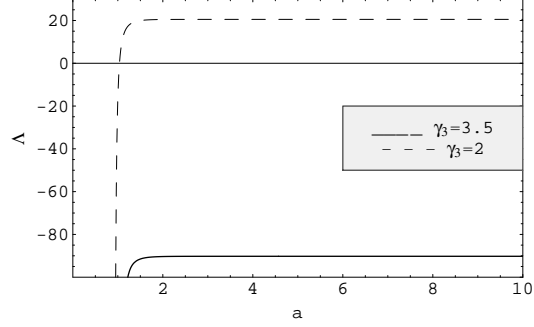


Fig.14

Fig. 13 shows the variation of G against $a(t)$ for different values of $\gamma_1 = 2, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_3 = 1$, $C = 1$, $B = 1$. Fig. 14 shows the variation of Λ against $a(t)$ for different values of $\gamma_1 = 2, 3, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_3 = 1$, $B = 1$, $C = 1$.

for $\Lambda \propto \frac{\ddot{a}}{a}$ or $\Lambda = \beta_3 \frac{\ddot{a}}{a}$, we have the corresponding parameters as, $\Omega_m^{\beta_3} = \frac{2(3-\beta_3)}{3(2-\beta_3-\beta_3\frac{p}{\rho})}$, $\Omega_\Lambda^{\beta_3} = \frac{-\beta_3(1+3\frac{p}{\rho})}{3(2-\beta_3-\beta_3\frac{p}{\rho})}$ and $\Omega_{total} = 1$. Now $\Omega_{total} = \Omega_m^{\beta_3} + \Omega_\Lambda^{\beta_3} = 1$ for all the models. Also we can compare these models by taking, $\Omega_m^{\beta_1} = \Omega_m^{\beta_2}$, so that $\beta_2 = \frac{3\beta_1}{(8\pi G + \beta_1)}$. Now we would like to take into account the present values of the density parameter and vacuum parameter obtained by the recent measurements. Considering $\Omega_{m0} = 0.33 \pm .035$, we calculate the present values of the proportional constants to be $1.7397K \leq \beta_1^0 \leq 2.3898K$, $1.905 \leq \beta_2^0 \leq 2.115$ and $3.7937 \leq \beta_3^0 \leq 4.2099$, where $K = 8\pi G_0$ and G_0 is the present value of the gravitational constant. Thus we get the value of β_3^0 to be lesser than the previous works [5, 14]. Again considering G to be time-dependent, we get the same values of the parameters as that with G constant, i.e., the ranges of $\gamma_1^0, \gamma_2^0, \gamma_3^0$ are same as that of $\beta_1^0, \beta_2^0, \beta_3^0$ respectively. Here also we get cosmic acceleration and the nature of variation G and Λ as well. We get two different cases regarding the variation of G and Λ . For the first two cases we see that G increases and Λ decreases with time, whereas for the third case G decreases and Λ increases with time. In all the cases the values become constant after a certain period of time, i.e., the present day values of G and Λ are constants. Thus these models with the phenomenological laws give us some interesting features of the cosmic acceleration and some modified values of the parameters. Also we get the natures of the Cosmological Constant and the Gravitational Constant over the total age of the Universe. We can also make use of the statefinder parameters to show the evolution of the Universe starting from radiation era to Λ CDM model.

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